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Numerical Analysis of Natural Convection in a Northlight Roof under Winterday Boundary Conditions

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Abstract— In this paper, numerical simulation is carried out to study the natural convection convective flows in a north light roof under winter day boundary conditions. The winter day boundary conditions like hot bottom wall due to room heating and cold ceiling due to environmental temperature is adopted for the present study. The steady state solutions have been obtained for a Rayleigh number ranging from $10^3$to 10^6 and $Pr = 0.7$. In this paper, natural convection heat transfer in a traditional north light roof is quantitatively investigated through isotherm pattern, stream line pattern, local and average Nusselt number. The results indicate that magnitude of the stream function is low at lower Rayleigh number due to conduction domination. However, it increases with increase in Rayleigh number due to transition from conduction dominant to convection dominated mode. It is observed that the rate of heat transfer is found to be minimum at the centre of the bottom wall. It increases further at a greater rate in the right hand side of the roof than left. This is because of large cooling area at the right side. It is noticed that as the Rayleigh number increases multiple cell solution is developed between hot bottom and cold inclined walls.

Index Terms — Natural convection, Roof, Heat transfer, Rayleigh number, Nusselt number.

1 INTRODUCTION

The mechanism of convective heat transfer and fluid flow in an enclosure has the center stage for many engineering applications. Some of these applications include floor heating, heat transfer from radiators and roofs, cooling of electronic devices, solar energy collectors, glazing windows, fire control in buildings. The roofs are the main parts of the building that protects the building from environmental effects such as rain, storm, particle matter in air and snow. The roofs of the building are built in such a way that, it reduces the heat loss from environment to room and vice-versa. Roofs of the buildings are constructed based on the climatic conditions, rain and architectural design. The rate of heat transfer and flow field inside the roof of the building is mainly depends on its shape and size. The geometry of roof can be different shapes such as gable, gambrel, saltbox, shed roof. Due to the winter day and summer day conditions, the natural convective currents are setup inside the roofs.

The buoyancy forces are induced because of the temperature difference between the ceiling of the room and sloping wall of the roof. The investigation of natural convection heat transfer and fluid flow inside the roof is important to analyze energy saving and to reduce heating or cooling load of buildings. In the present study, natural convection in different roofs of building thoroughly made. The literature on the transitional and laminar natural convection in closed cavities is reviewed by Fusegi and Hyun [1]. The complexity in fluid flow and heat transfer with realistic boundary conditions was carried out. Asan and Namli [2, 3] studied the energy transfer and fluid flow caused by density difference in a gable roof under winter and summer day boundary conditions the comparison has been made between summerday and winterday boundary conditions. It is found that the overall heat transfer is mainly depends on aspect ratio and Rayleigh number for winter day boundary conditions. However, for summer day boundary conditions Nusselt number is independent of Rayleigh number. Varol et al. [4] carriedout the computations on natural convective flows in a salt box roof for both summer like and winter like boundary conditions. The results are compared values of gable roof for same heating length bottom wall. It is found that lower heat transfer is obtained, when salt box type roofs are used. It is noticed that the energy rate is becomes more in winterday like boundary conditions than that of summerday boundary conditions. Varol et al. [5] studied steady state natural convection in a Gambrel roof for both summerday and winterday boundary conditions. It is found that winterday boundary conditions have more influence on heat transfer than summer day due to natural convection mechanism. Also, it is observed that energy transfer is lower than that of Gable roof for same inclination of side wall and for same Rayleigh number. The computations on natural convective fluid flow in shed roofs with and without eave of buildings for summer and winter seasons have been carried out by Koca et al. [6, 7]. The reported results indicate that eave length and aspect ratio are the more significant parameters on heat transfer for the same Rayleigh number. Recently, a detailed review of literature on natural convective heat transfer in an attic shaped space carried out by Saha and Khan [8]. This paper focused on significant number of recent studies on topics related to energy transfer and flow in attic shaped space. Sigrid et al. [9] have performed the evaluation and optimization of a traditional north light roof. Also, the industrial plant energy consumption for an optimized north-light roof shape is studied. However, the attempt on heat transfer and fluid flow is not dealt in this study.

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Nomenclature

g	acceleration due to gravity, m s^{-2}	X	dimensionless distance along x co-ordinate
k	thermal conductivity, W m^{-1} K^{-1}	Y	dimensionless distance along y distance
H	height of the north light roof with reference to central vertical line of the bottom wall, m	*Greek symbols*	
		α	thermal diffusivity, m^2 s^{-1}
L	length of the bottom wall, m	β	volume expansion co-efficient, K^{-1}
Nu	local Nusselt number	θ	dimensionless temperature
$\overline{Nu}$	average Nusselt number	υ	kinematic viscosity, m^2 s^{-1}
P	dimension less pressure	ρ	density, kg m^{-3}
Pr	Prandtl number	ψ	stream function
Ra	Rayleigh number	*Subscripts*	
T	temperature, K	b	bottom wall
U	x component of dimensionless velocity	c	cold wall
V	y component of dimensionless velocity	h	hot wall

From the observation in the thorough literature on heat transfer and fluid flow analysis in different shapes like gable, gambrel, salt box and shed roof is carried out till date. The literature clearly shows that there is a void existing in the analysis heat transfer and fluid flow in north light roof. The numerical simulation of north-light roof is much essential than other roof due to the following reasons. The north light roof will bring more sun light inside the working environment from morning to evening. The energy consumption in a building, one which is having north-light roof is 54% lesser than that of a conventional flat roof [9].
The contribution of present work is mainly concentrating on the influence of heat transfer with winter day boundary condition in a north light roof. It is noticed in the literature review that, the computations are carried out on optimizing the shape of the north light roof for the thermal management. However the effects of winter day boundary conditions have not been dealt. Thus, the present study is providing the detail information on natural convection in a north light roof for wide range of parameters.

II. Mathematical Model

The configuration of the physical domain selected for the analysis is shown in Fig. 1. It is assumed that roof is filled with Newtonian, incompressible, viscous fluid.

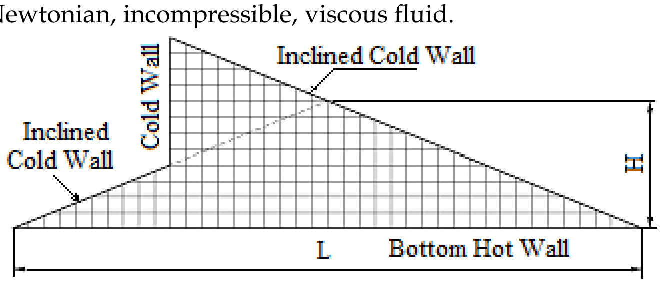


Fig. 1. The physical domain of the system.

During the energy transfer, variation of thermo fluid physical properties of the working fluid is assumed to be constant. The Boussinesq approximation is invoked for buoyancy term. The gravity forces are acting vertically downward. The differential equations such as conservation of mass, momentum and energy are governing the natural convection inside the physical domain. The following are change of variables used to reduce the governing equation to non-dimensional form;

$$X = \frac{x}{L} \text{ (1)},\ Y = \frac{y}{L},\ U = \frac{uL}{\alpha},\ V = \frac{vL}{\alpha},\ \theta = \frac{T - T_C}{T_h - T_C} \quad (1)$$

$$P = \frac{pL^2}{\rho\alpha^2},\ \Pr = \frac{v}{\alpha},\quad Ra = \frac{g\beta(T_h - T_C)L^3 \Pr}{v^2}$$

The following are the non-dimensional form of governing equations.

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (2)$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \Pr\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \quad (3)$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \Pr\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + Ra\Pr\theta \quad (4)$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \quad (5)$$

The boundary conditions for the physical model depicted in Fig.1 are as follows:
In this model u = v = 0 for all the walls,
On inclined and vertical walls, T = Tc
At the bottom wall, T = Th.

III. Evaluation of Stream Function and Nusselt Number

1. Stream Function

The fluid motion is displayed using stream function ψ obtained from the velocity component U and V. The relation between ψ and velocity components (Batchelor [10]) for two dimensional flows is;

$$\frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} = \frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \quad (6)$$

Using the above definition of the stream function, the motion of the fluid in anti-clockwise circulation and clockwise circulation are represented by positive and negative sign respectively.

2.Nusselt number

The coefficient of heat transfer is representing local Nusselt number (Nu) and is defined by;

$$Nu = -\frac{\partial \theta}{\partial n} \quad (7)$$

Where, n denotes the normal direction on a plane.
The average Nusselt number are obtained by integrating local Nusselt number along the wall as follows,

$$\overline{Nu} = \frac{1}{L}\int_0^L Nu_x \, dX \quad (8)$$

IV.NUMERICAL TEST

In the present study a set of governing equations are integrated over the control volume, which produces a set of algebraic equations. The Pressure-Implicit with splitting of operators (PISO) algorithm developed by Issa [11] is used to solve the coupled system of governing equations. The set of algebraic equations are solved sequentially using alternating direction implicit method (ADI) [12]. A second order upwind differencing scheme is used for formulation of convection contribution to the coefficient in finite volume equations. The central differencing scheme is used to discrete the diffusion terms. The computation is terminated when all the residuals are less than 10-5.

Average Nusselt Number ($\overline{Nu}$)	**Rayleigh Number (*Ra*)**			
	10^3	**10^4**	**10^5**	**10^6**
$\overline{Nu}$ (Ref. [3])	4.87	5.12	7.15	12.27
$\overline{Nu}$ (Present)	4.79	5.13	7.08	12.26

Table I.Comparision of Average Nusselt Number for Gable Roof with Aspect Ratio = 1.

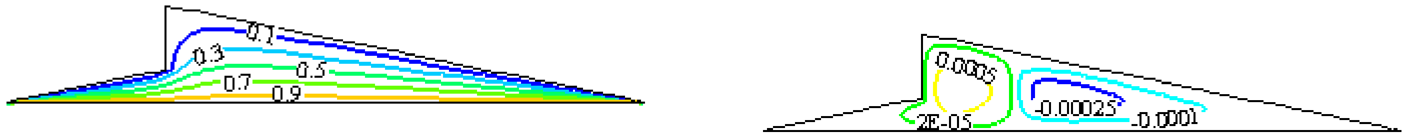
The computation domain is discretized in to a number of sub domains starting from 31x31, 41x41, 51x51, 61x61, 71x71, and 121 x 121. It may be noted that the computational grid in north-light roof is generated normal to the bottom heating wall. The grid tests are made for the 31 x 31 to 121 x 121 to obtain the optimal grid dimensions. It is observed that the grid size 61x61 and above shows that there is no appreciable changes in the average Nusselt number in comparison with literature Asan and Namli [3]. In the view of this, 61x 61 grid is selected for the further computation.

In order to validate the predictive compatibility and accuracy of the present methodology, computations are preferred using the configuration and boundary conditions of Asan and Namli [3]. The finite volume method is preferred to study the numerical simulation of buoyancy flow in a gable roof under winter day boundary conditions. The comparison has been made with mean Nusselt number for AR= 1 including high Rayleigh numbers. The results are presented in table 1. It is observed that there is a good agreement between Asan and Namli [3] and the present study with maximum discrepancy of 1.6%. The agreement is found to be excellent which validates the present code.

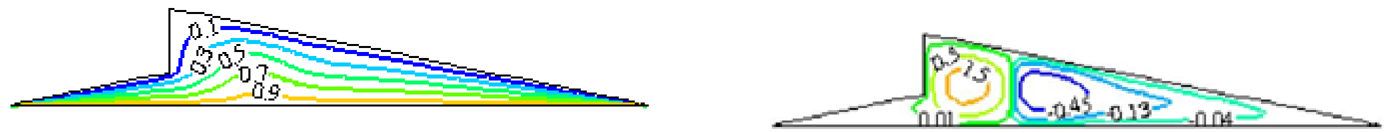
V. RESULTS AND DISCUSSION

1. Effect of Rayleigh Number; Aspect Ratio

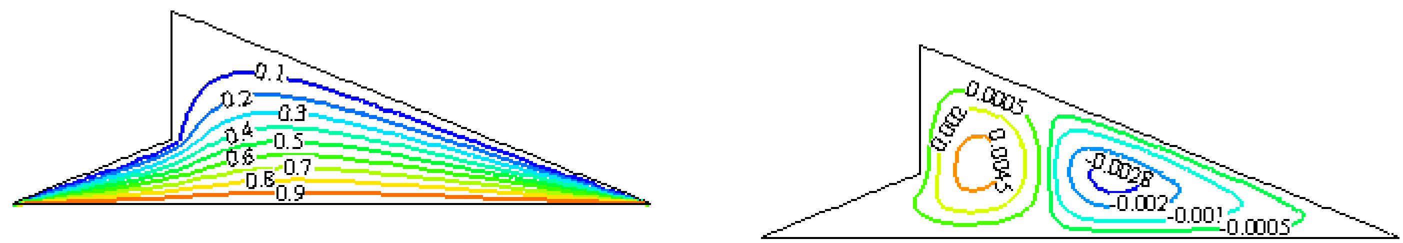
The numerical simulations are carried out to obtain the heat transfer and flow field inside a north light roof to cold climate condition due to buoyancy forces. The influence of Rayleigh number and aspect ratio (H/L) = 0.1 and 0.2 on heat and fluid flow with stream function, temperature contours, local and average Nusselt number is discussed. Figs. 2(a) - (f) illustrates the isotherms and stream lines of the numerical results of various Ra = 10^3 – 10^6, H/L = 0.1 - 0.2 and Pr = 0.7 for winter day boundary conditions. As expected, the fluid rises up from slightly left side with reference to the mid portion of the bottom hot wall. This is because of unsymmetrical geometry and large cooling area in the left side of the north light roof than the other end. The fluid gets cooled at the top and flow down along the inclined and vertical walls forming two rolls with clockwise and anticlockwise rotation inside the cavity. It is observed from the Fig. 2(a) and 2(b) that at Ra = 10^3 - 10^4, the magnitude of the stream function are very low and the heat transfer is primarily due to conduction. During this conduction dominant heat transfer, the isotherms are smooth curves which span the entire enclosure and $\theta \geq 0.7$ are symmetric with respect to vertical center line. The temperature contours as indicated in Fig. 2 (a) and 2(b) remains invariant up to Ra = 10^4. For Ra > 10^4, the circulations near the central regime are stronger and plume like thermal distribution is obtained. The isotherms $\theta \geq 0.7$ starts getting shifted towards the cold walls as depicted in Fig. 2(c) to 2(f) for Ra = 10^5 – 10^6 and H/L = 0.1 to 0.2. The presence of significant convection is also exhibited in temperature contours which start getting deformed and pushed towards the cold inclined walls. As Ra increases from 10^4 to 10^6, the buoyancy had driven convective current inside the north light roof increases as seen from the greater magnitude of stream functions [Fig. 2(c) to 2(f)]. The circulations are greater at the center and least at the walls due to no slip boundary conditions. The greater circulations are occurred in the left and right side in a north light roof with respect to centre vertical line. The magnitude of the circulations is increased with increase of Ra and greater at the centre of the circulations. The isotherms are get compressed to cold inclined walls, which leads to a small temperature gradients near the bottom hot and inclined cold walls. It is observed that for Ra = 105 and 106 [Figs. 2(d) - (f)] the thermal boundary layer is developed almost throughout the entire cavity.
Comparison of Figs. 2(e) and 2(f) shows that as aspect ratio of north light roof increases from 0.1 to 0.2, the values of isotherms and stream functions in the core north light roof increases. The isotherms are concentrated to dragging towards the vertical cold wall due to enlarged space. As the aspect ratio increases to 0.2, this trend is increased up to $\theta \leq 0.6$ and becomes dense at the middle of the right inclined cold wall. For $\theta \geq 0.8$ are symmetric about vertical central line. The magnitudes of the stream functions increase by almost 8 times as the aspect ratio of the north light roof increase from 0.1 to 0.2. The multiples cells are set below the left cold vertical walls for aspect ratio 0.2, higher values of stream function setting in the left side than right side. It clearly indicates that the transition from two vortex solution to multiple vortexes depends on Rayleigh number and aspect ratio. The convection is more dominated in north light roof of aspect ratio 0.2 than 0.1.



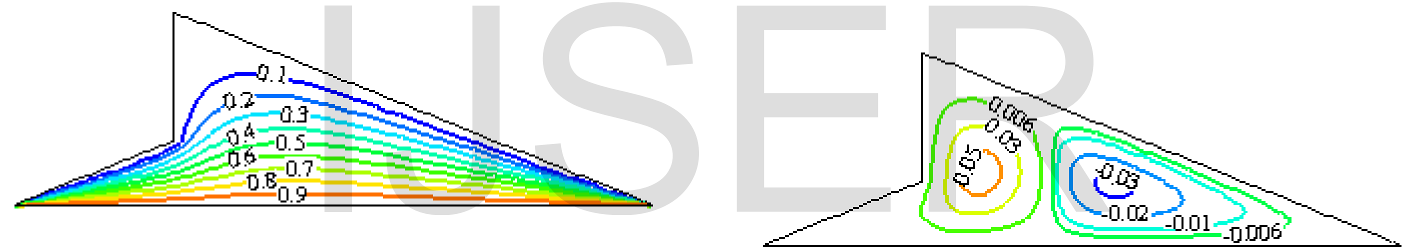
a) Ra = 10^3, H/L = 0.1



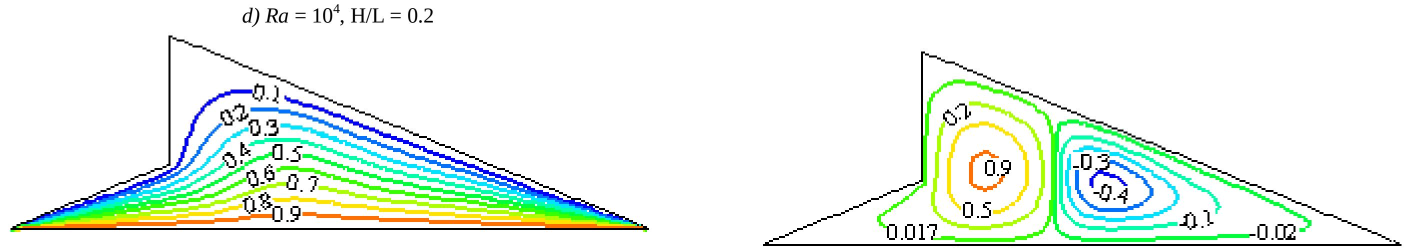
b) Ra = 10^6, H/L = 0.1



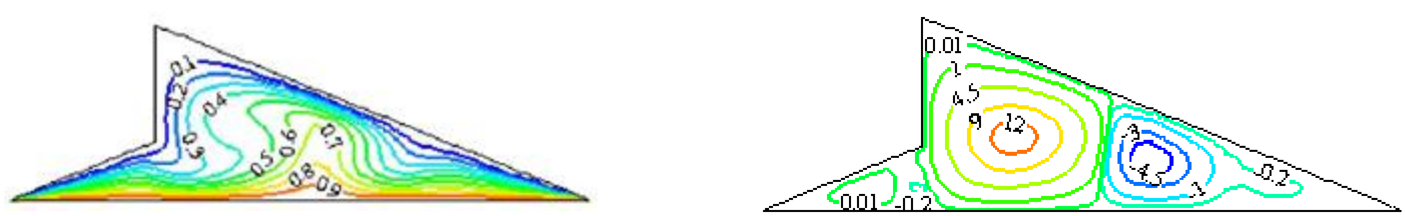
c) Ra = 10^3, H/L = 0.2



d) Ra = 10^4, H/L = 0.2



e) Ra = 10^5, H/L = 0.2



f) Ra = 10^6, H/L = 0.2

Fig. 2 Variation of Isotherms(left) and Stream functions(right) for different aspect ratio and Ra (103-106)

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2. Heat Transfer Rate; Local and Average Nusselt Numbers

Fig. 3 shows the effect of Ra for AR = 0.2 on the local Nusselt numbers at the bottom hot wall. The heat transfer rate is very high at the right side of the north light roof for all the Ra. This is

because of inclined cold wall is very close to hot wall and the span is larger than the left side. The local Nusselt numbers reduces towards the center of the hot wall for the given Rayleigh numbers. As expected the heat transfer rate is very high at the corners, because of the intersection of inclined cold wall and bottom hot wall is occurred. The local Nusselt numbers are same for Ra = 103 and 104, due to conduction dominated heat transfer. For Ra =105, the local Nusselt number variations are increasing and decreasing trend and minimum occurred at a distance X = 0.58. The heat transfer rate is slightly higher than for Ra ≤ 104. The physical reason for this is the right central cells dragging towards the right corner. For Ra = 106, the left bigger cells are extending towards the left side of the bottom hot wall. Hence the local Nusselt number is increasing, decreasing and minimum occurred at the centre of the hot wall. The Nu steadily increased at X = 0.5 to 0.7 and decreased up X = 0.8 and then increased. This is because of the isotherms are compressed to centre of the right inclined wall [Fig. 2(f)]. The magnitude of the stream functions at the left side is very high.The variations of the Nu for different AR and Ra = 105 is shown in Fig. 4. It is observed that the Nu is decreasing up to X ≥ 0.5 for all the AR. However the minimum value of the Nu is decreased and it is shifting to left side as the AR increased. The heat transfer rate is increased with increase of AR for X ≥ 0.5. The overall effect of heat transfer rates are shown in Fig. 5. The distribution of average nusselt number at hot bottom wall is plotted versus logarithmic Ra. The average nusselt number is increasing monotonically with increase of Ra. It is noticed that it remains constant up to Ra = 10^4 due to dominant heat conduction mode for the given AR. The convective dominated heat transfer is occurred for Ra ≥ 10^4, the AR considered for the investigation. It is observed that the changes in average nusselt number is more pronounced for higher AR than the lower one.

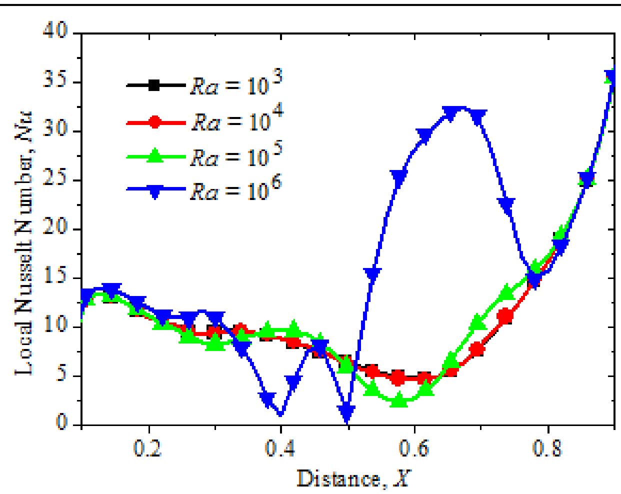

Fig.3 Variation of local Nusselt number at bottom wall for Aspect Ratio (H/L) = 0.2

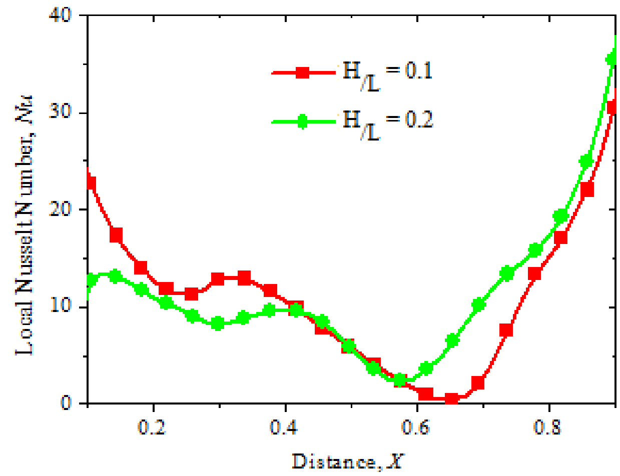

Fig.4 Variation of local Nusselt number at bottom wall for Ra = 10^5.

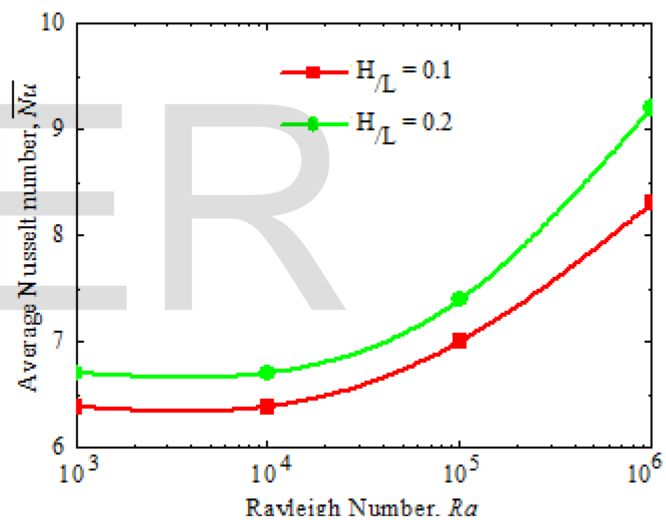

Fig.5 Variation of average Nusselt number at bottom wall for different aspect ratio

VI. Conclusions

The paper has reported the numerical results of laminar, steady, 2D natural convection in a north light roof for winter day conditions. The following observations have been made,

a) The dominated conductive heat transfer is observed up to Ra = 10^4 for all AR under consideration.

b) The *Nu* is increased monotonically with increase of Ra and thermal and flow fields are affected by the shape and AR of the geometry.

c) The heat transfer rate increases with increase of AR for all Ra as computed.

d) It is noticed that as AR increases, minimum value of Nu is shifting towards the centre of the bottom heating wall.

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REFERENCES

[1] Fusegi T, Hyun J.M, "Laminar and trasitional natural convection in an enclosure with complex and realistic conditions", Int. J. Heat Fluid Flow 15 (4), 1994, pp. 258-267.

[2] Asan H, Namli L, "Laminar natural convection in a pitched roof of traingular cross-section; summerday boundary conditions", Energy and Buildings, 33, 2000, pp. 69-73.

[3] Asan H, Namli L, "Numerical simulations of buoyant flow in a roof of triangular cross-section under winterday boundary conditions", Energy and Buildings, 33, 2001, pp. 753-757.

[4] Yasin Varol, Ahmet Koca, Hakan F, Oztop, "Laminar natural convection in salt box roof for both summerlike and winterlike boundary conditions", J. of Applied sciences, 6 (12) , 2006, pp. 2617-2622.

[5] Yasin Varol, Ahmet Koca, Hakan F, Oztop, "Natural convection heat transfer in Gambrel roofs", Building and Environment, 42, 2007, pp. 1291-1297.

[6] Hakan F, Oztop, Yasin Varol, Ahmet Koca, "Laminar natural coinvection heat transfer in a shed roof with or without eave for summer season", Applied Thermal Engineering, 27, 2007, pp. 2252-2265.

[7] Ahmet Koca, Hakan F, Oztop, Yasin Varol, "Numerical analysis of natural convection in shed roofs with eave of buildings for cold climates", J. Computers and Mathematics with Applications, 56, 2008, pp. 3165-3174.

[8] Suvash. C.Saha, Khan M M K, "A Review of natural convection and heat trasnfer in attic-shaped space", Energy and Buildings, 43, 2011, pp. 2564-2571.

[9] Sigrid A, Hao Liu, Mariam Wahed, Quianchuan Zhao, "Evaluation and optimization of a tradetional north-light roof on industrial plant energy consumption", J. of Energies, 6, 2013, pp. 1944-1960.

[10] Batchelor G K, An Introduction to Fluid Dynamics, cambridge university press, 1993.

[11] Issa R I, "Solution of implicitly discretised fluid flow equations by operator splitting " , J. Comput. Phys. Vol. 62, 1985, pp.40-65.

[12] Roache P J, Computational Fluid Dynamics, Hermosa, Albuquerque, New Mexico, 1982.